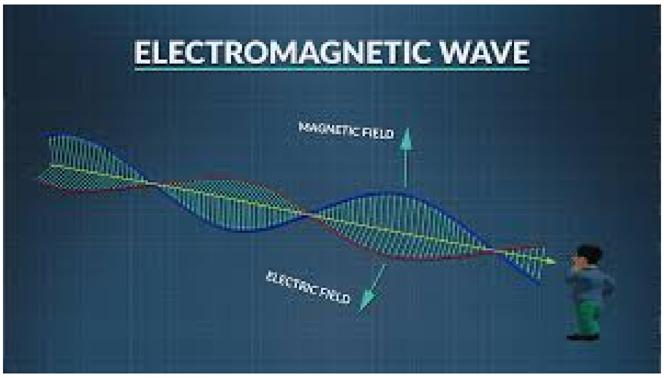
ELECTROMAGNETIC THEORY
UNIT - II (A), B.Sc II.



BY : DR. MANOJ SINGH

Unit 2

Page No.
Date : / /201

Electro magnetic Induction

[redacted] Electromagnetic Induction, Maxwell equations and Betatron.

Introduction

When no. of Magnetic field lines passing through a circuit changes with time, an EMF is set up in the circuit. If circuit is closed then a current starts flowing in the circuit. The emf and current so produced are called Induced emf and Induced current & This phenomenon is called EMI.

Define Faraday's Law of Electromag. Indn.?

(3) Faraday's Law

In 1831, Michael Faraday published his two famous laws. These are -

First Law : When Magnetic flux through the circuit changes rapidly, induced emf b/w end terminals is propotional to negative rate of change in magnetic flux.

i.e. $\varepsilon \propto -\dfrac{d\phi_B}{dt}$

$$E = -k\frac{d\phi_B}{dt}$$

here k = newmann's constant its value is unity.

i.e $\boxed{E = -\dfrac{d\phi_B}{dt}}$ — (1)

If coil has N Turns then induced emf

$$\boxed{E = -N \frac{d\phi_B}{dt}} \quad OR \quad \boxed{E = \frac{-d}{dt}(N\phi_B)}$$

here $N \cdot \phi_B$ = no. of Magnetic flux linkage
= total Mag. flux

This law is also known as Newmann's law.

Second law : "The direction of induced emf & induced current is such that they oppose that cause which produced it"
Here main cause is relative motion b/w magnet and the coil.
This law is also called Lenz law, based on conservation of energy

Differential form of Faraday's Law -

We have induced emf $E = \frac{-d\phi_B}{dt}$ — (1)

Now surface integral of $\vec{B}$ is the magnetic flux.

$$\boxed{\phi_B = \oint \vec{B} \cdot d\vec{s}}$$

Now As flux changes, electric field is induced within the circuit.
Its line integral gives induced emf.

2

note

$$\boxed{\mathcal{E} = \oint \vec{E} \cdot d\vec{l}}$$

Putting ϕ_B & $\mathcal{E}$ in eq^n (1)

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \oint \vec{B} \cdot d\vec{s}$$

3

If circuit is fixed, magnetic field change by magnet, $\frac{d}{dt}$ can be enter inside the integral.

$$\oint \vec{E} \cdot d\vec{l} = -\oint \frac{\partial}{\partial t} \vec{B} \cdot d\vec{s} \quad \text{——— (2)}$$

Using Stoke's theorem

$$\oint_l \vec{E} \cdot d\vec{l} = \oint_s \text{curl } \vec{E} \cdot d\vec{s} \quad \text{——— (3)}.$$

On comparing eq^n (2) & (3).

$$\boxed{\text{curl } \vec{E} = \frac{-\partial \vec{B}}{\partial t}}$$

∇- operator

Or $$\boxed{\nabla \times \vec{E} = -\frac{\partial B}{\partial t}}$$

This is required form of Faraday's law

curl of Electrostatic field is zero i.e curl But curl of Induced Electric field which produced by time varying magnetic fi is never be zero.

Modifications in Ampere Circuital Law

Ampere circuital law is not applicable to those circuits which contain capacitor. because there is no actual flow of electrons takes place b/w the plates of capacitor during charging or discharging.

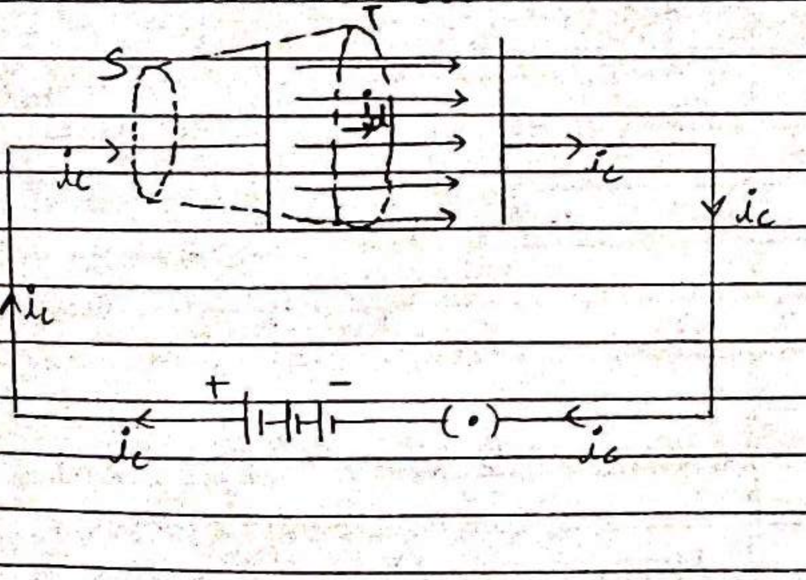
We take following circuit — in which there are 2 curves S and T attached together. Apply A.C.L to the loop S. 4

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \cdot i \quad \text{——— (1)}$$

and apply A.C.L to the loop T dig.

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \times 0 = 0 \quad \text{——— (2)}$$

Curve S and T are close and attached so both eq^n should give same result so it is a contradiction.



To avoid this contradiction, Maxwell gave a new concept of displacement current i_d. It is generated by time varying Elec. Field.

Time varying E.F. b/w plates

$$E(t) = \frac{\sigma(t)}{\varepsilon_0}$$

$$\sigma(t) = \frac{q(t)}{A}$$ 5

$$E(t) = \frac{q(t)}{A\varepsilon_0}$$

$$\frac{dE(t)}{dt} = \frac{1}{A\varepsilon_0}\frac{d}{dt}q(t)$$

$$\frac{dq(t)}{dt} = i_d = \text{displacement current}.$$

$$\frac{dE(t)}{dt} = \frac{i_d}{A\varepsilon_0}$$

$$\boxed{i_d = A\varepsilon_0 \cdot \frac{dE(t)}{dt}}$$

Now eq^n ② will be

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 i_d \quad \text{———} ③$$

Now, we have to show

$$i_d = i_c$$

We know that rate of charge flow in a wire is called conduction current

i-e $i_c = \frac{dq}{dt}$ ——— (4)

Taking expression of displacement current

$$i_d = A\varepsilon_0 \cdot \frac{dE(t)}{dt}$$

$$i_d = \varepsilon_0 \frac{d[E(t) A]}{dt}$$

$$\because E(t) \times A = \phi_E .$$

$$i_d = \varepsilon_0 \frac{d}{dt} \phi_E(t) .$$

By Gauss law

$$\phi_E = \frac{q}{\varepsilon_0}$$

$$i_d = \varepsilon_0 \frac{d}{dt} . \left(\frac{q}{\varepsilon_0}\right)$$

$$i_d = \frac{dq}{dt} \quad \text{[from (4)]}$$

$$\boxed{i_d = i_c}$$

state maxwell eqn in integral & [illegible] form [illegible] explaining [illegible] their physical significance

Type your te

Now, eqn ① & ③ gives the same result and modified form of ACL

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 (i + i_d)$$

$$= \mu_0 \left(i + A\,\varepsilon_0 \frac{dE}{dt}\right)$$

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 \left(i + \varepsilon_0 \frac{d\phi_E}{dt}\right)$$

Write M.E. of electromagnetism?

Maxwell Equations

In 1862, Maxwell formulated the basic laws of electricity and magnetism in the form of four fundamental equations known as Maxwell equations. These are –

Maxwell first equation: It is Gauss law in electrostatics. Acc. to it "Electric flux passing through any closed surface is equal to $\frac{1}{\varepsilon_0}$ times to the net charge enclosed by that surface."

i.e.

$$\oint \vec{E}\cdot\vec{ds} = \frac{q}{\varepsilon_0}$$

Now, single electric charge can exist

Maxwell second equation: It is Gauss law in Magnetism. Acc. to it – "Magnetic flux passing through any closed surface is equal to zero".

i.e. $\oint \vec{B}\cdot\vec{ds} = 0$

means single magnetic pole does not exist

Maxwell Third Equation: It is Faraday's law of Electromagnetic Induction.
Acc. to it — "Induced emf in the circuit is equal to negative time rate change in Magnetic flux."

i.e. $\oint \vec{E}\cdot\vec{dl} = -\dfrac{d\phi_B}{dt}$.

Maxwell Fourth Equation: It is modified form of ACL. It signifies that varying Elec. field produce magnetic field.

i.e. $\oint \vec{B}\cdot\vec{dl} = \mu_0\left(i + A\varepsilon_0\dfrac{dE}{dt}\right)$

Above equations are in integral form they can express in differential form also.

$$\text{div}\,\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\text{div}\,\vec{B} = 0$$

$$\text{curl}\,\vec{E} = \frac{-\partial B}{\partial t}$$

$$\text{curl}\,\vec{B} = \mu_0\left(\vec{J} + \varepsilon_0\frac{dE}{dt}\right)$$

$\text{div}\,\vec{E} = \dfrac{\rho}{\varepsilon_0}$

$\text{div}\,\vec{B} = 0$

$\text{curl}\,\vec{E} = -\dfrac{\partial B}{\partial t}$

$\text{curl}\,\vec{B}$ $= \mu_0(\vec{J} +$

II^nd unit ... and working of a betatron?

Betatron 100%

It is a device which is used to accelerate electrons to higher energy upto 100 Mev To 500 Mev. It was invented by W. Kerst in 1942.

Principle - It is based on two facts -

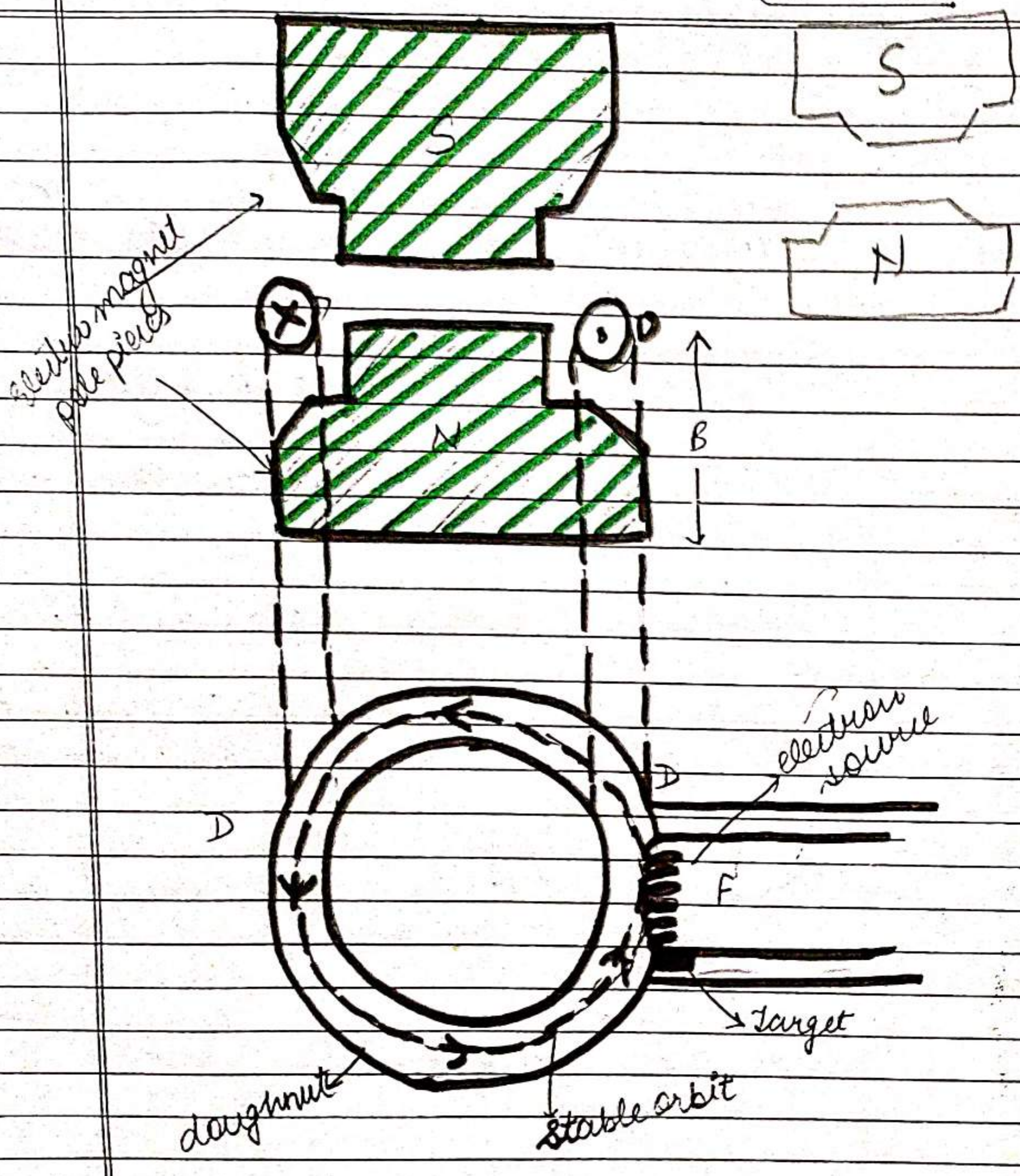
When a charge particle enter in magnetic field, it experience Lorentz force.

Phenomenon of electromagnetic induction.

In this device electron revolve in stable orbit not take spiral path as in cyclotron.

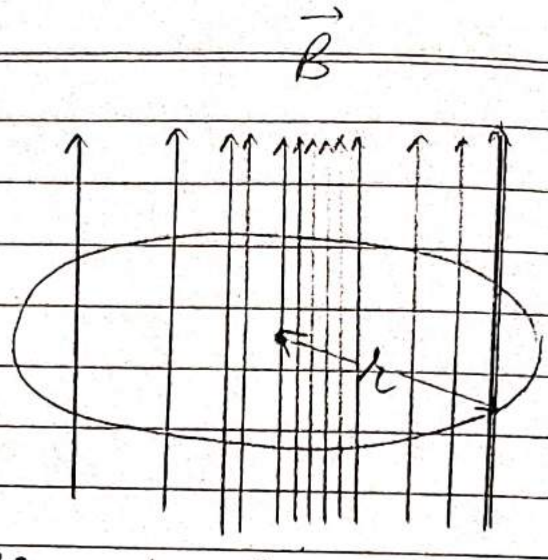
Construction: It contain doughnut shaped evacuated chamber placed between two opposite poles of an electro magnet and alternating current is used in the formation of electromagnet.

: Explain the principle of working of a betatron. Derive the condition for its operation. How is it achieved?



The shape of magnets is such that it produce increasing magnetic field towards the center of the doughnut as shown in fig.

Page No.
Date: / /20



Condition for Betatron.

When an electron revolve in the stable orbit rate of increament of magnetic flux.

Its negative value gives induced emf.

$$E' = \frac{-d\phi}{dt}$$

This induced emf perform work on electron.

W = charge x potential differ. or induced emf

$$W = -e \times \frac{-d\phi}{dt}$$

$$W = e\frac{d\phi}{dt} \quad \text{———} (1).$$

If f is the force developed on electron by induced emf then work done by this force

$$W = f \times 2\pi r \quad \text{———} (2)$$

Both work should be equal

$$F \times 2\pi r = e\frac{d\phi}{dt}$$

$$\boxed{F = \frac{e}{2\pi r}\frac{d\phi}{dt}}$$

This force increase kinetic energy of electron and electron tends to increase its radius but Lorentz force provide necessary centripetal force.

$$qvB = \frac{mv^2}{r}$$

$$qB = \frac{mv}{r}$$

$$qB = \frac{P}{r}$$

$$P = qBr$$

Acc. to Newton's law, $F = \frac{dP}{dt}$

$$F = \frac{d}{dt} qBr$$

$$= qr\frac{dB}{dt}$$

Page No.
Date : / /201

$$\boxed{F = e r \frac{dB}{dt}}$$

For stable orbit both expression of force should be equal.

$$\frac{e}{2\pi r}\frac{d\phi}{dt} = e r \frac{dB}{dt}$$

$$\frac{d\phi}{dt} = 2\pi r^2 \frac{dB}{dt}.$$

Or. $d\phi = 2\pi r^2 dB$

On Integration

$$\int_0^{\phi} d\phi = \int_0^{B} 2\pi r^2 dB.$$

$$\boxed{\phi = 2\pi r^2 B}$$

This is the necessary Condition for stable orbit of electron in Betatron which is -

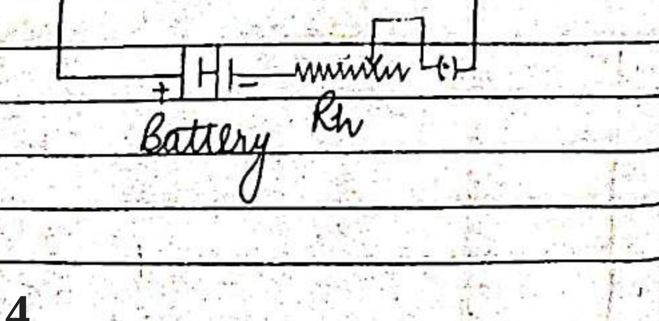
"Double flux is required to make orbit stable in comparison of normal shaped magnet".

Working :→ Electron/from electron gun is injected in doughnut at that moment when mag. field increases in upward direction so, electron get energy by induced emf and its kinetic energy increases as flux is double there so, Lorentz force kept its orbit stable and it hit the target when necessary.

Self & Mutual Induction

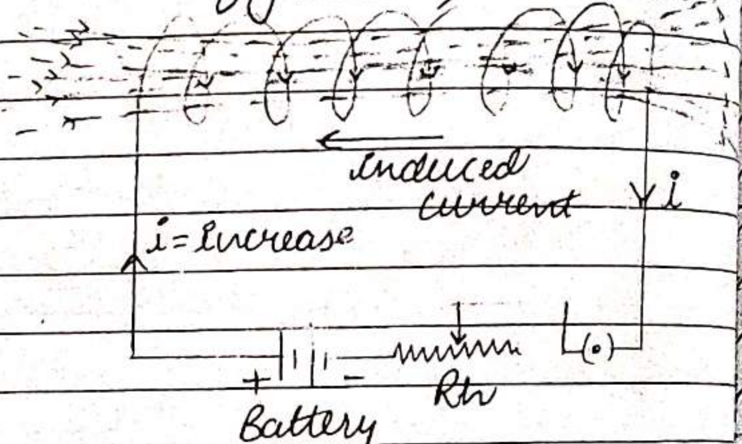
What is Self Induction. Write the coefficient of self-Induction ?

Self Induction / Inertia of current
When a current passed through a coil, it produces a magnetic flux which always linked with the coil. (fig a)

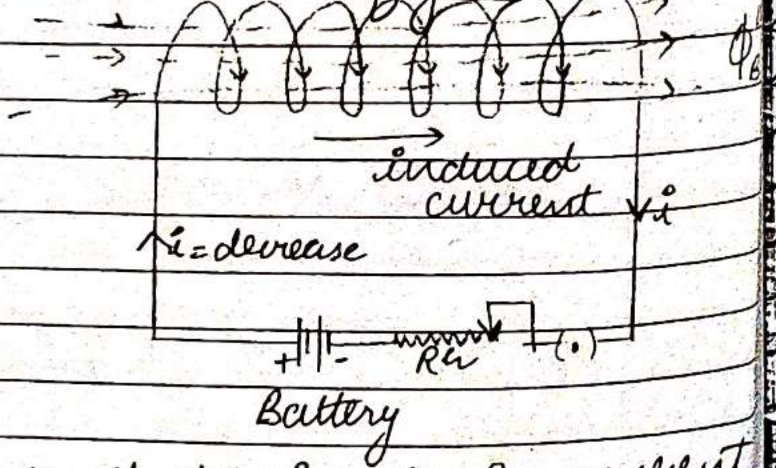


Rheostat (धारा नियंत्रक)

when the value of current is increased by Rheostat then the flux linked wit the coil is also increased due to wh an induced or back emf is set up in the circuit and induced current produ in the direction (same) opposite to main curre It is shown in fig (b)



when the value of current is decreased Rheostat then the flux linked with th coil is also decreased due to which an induced or back emf is set up in the circuit and induced current prod in the same direction as to main current. It is shown in fig (c)



This phenomenon of opposing main current by a coil is called self induction

Eg: choke coil based on this phenomenon.

• Self Inductance. (L)

let N = no. of turns in coil

ϕ = flux linked with each turn.

$N\phi$ = total magnetic flux / no. of magnetic flux linkages.

Experimentally $N\phi \propto i$

or $N\phi_B = Li$

$$L = \frac{N\phi_B}{i} \quad \text{——— (1)}$$

L is called self inductance of the coil.

when this flux change with current induced emf $E = -N \cdot \frac{d\phi_B}{dt}$

$$E = -\frac{d(N\phi_B)}{dt}$$

Put $N\phi_B = Li$

$$E = \frac{-d(Li)}{dt}$$

$$E = L\left[\frac{-di}{dt}\right]$$

$$\boxed{L = -\frac{E}{\frac{di}{dt}}} \quad \text{——— (2)}$$

Page No.
Date : / /201

We also know that the work is also done by battery to increase or decrease the current.

$$W = \frac{1}{2} L i_0^2$$

$$\boxed{L = \frac{2W}{i_0^2}} \quad \text{——— (3)}$$

Definition of Self Inductance

From eqn (1), it is numerically equal to the number of magnetic flux linkages attach with the coil when unit current is passed through it.

From eqn (2), it is numerically equal to the back emf of the coil when the rate of change of current in the coil is unity.

From eqn (3) it is numerically equal to the twice of the work done against induced current in establishing unit current in the coil.

Unit of Self Inductance

As $L = \frac{-E}{\frac{di}{dt}}$

Page No.
Date : / /201

$$\text{unit} = \frac{\text{volt}}{\left(\frac{\text{amp}}{\text{sec}}\right)}$$

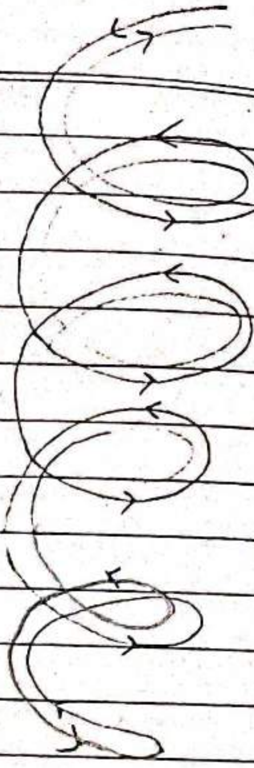
$$\frac{1\,V}{\frac{1\text{ amp}}{\text{sec}}} = 1 \text{ henry} = 1H$$

"The self inductance of a coil will be 1 henry when an induced emf of 1 volt is set up in the coil due to a current changing at the rate of 1amp/ sec in the coil".

* Non-Inductive Binding of Resistance

A resistor is normally a resistance wire wound on a wooden cylinder but it has few inductance also means flux linked with it.
To minimize flux we wound a wire by making it double then each and every point two currents lie in opposite direction so linked flux becomes zero. Now it becomes non-inductive resistance.

Ques What do you mean by self-inductance. Obtain an expression for self-inductance of a long solenoid?



Ques Determine self-inductance for—

a) Solenoid b) Toroid.

7.5m (a) SOLENOID: let us consider a long air-core solenoid having length 'l' and total no. of turns 'N' with cross-section area 'A'. Let a current i flow through it then magnetic field induction inside the core $B = \mu_0 n i$

put $n = \frac{N}{l}$

$$B = \mu_0 \frac{N}{l} i$$

The flux linked with each turn

$$\phi_B = BA = \mu_0 \frac{N}{l} i \times A$$

Page No.
Date : / /201

Flux linked with whole solenoid

$$N\phi_B = \mu_0 N = \frac{\mu_0 N^2 i A}{l}$$

By definitation of self inductance

$$L = \frac{N\phi_B}{i}$$

put $N\phi_B$

$$\boxed{L = \frac{\mu_0 N^2 A}{l}} \quad H$$

This is required expression for air core solenoid.

[illegible]

If solenoid is wound on a core, having permeability μ ($\mu = \mu_r \mu_0$) then the ;

$$\boxed{L = \frac{\mu N^2 A}{l}} \quad H$$

If material is ~~diem~~ diamagnetic then ~~μ_0~~ ~~∞~~ L value decrease.
If material is paramagnetic then L value increase.

20

b) TOROID : Let us consider an endless solenoid of radius 'r' have N turns carrying a current i then magnetic field in its core

$$B = \mu_0 n i$$

put $n = \frac{N}{l}$

$$B = \mu_0 \frac{N}{l} i$$

again $l = 2\pi r$

$$B = \frac{\mu_0}{2\pi} \frac{Ni}{r}$$

Flux linked with each turn

$$\phi_B = \left(\frac{\mu_0}{2\pi} \cdot \frac{Ni}{r} \right) A$$

Total fluxed linked with Toroid

$$N\phi_B = \frac{\mu_0}{2\pi} \frac{N^2 i}{r} \cdot A$$

By definition of self inductance

$$L = \frac{N\phi_B}{i}$$

$$\boxed{L = \frac{\mu_0}{2\pi} \frac{N^2 \cdot A}{r}} \; H.$$

This is req. expression for L for air core Toroid

If anchor ring is present with permeability μ ($\mu = \mu_r \cdot \mu_0$) :

$$\boxed{L = \frac{\mu N^2 A}{2\pi r}} \ H$$

Ques Calculate self inductance of a coil having 100 turns. If a current of 1 ampere produce a Mag ~~Field~~ flux of 5×10^{-5} Weber through the coil ?

2016

$N = 100$ turns

$\phi_B = 5 \times 10^{-5}$

$i = 1$ amp.

$$L = \frac{N \phi_B}{i}$$

$$L = \frac{100 \times 5 \times 10^{-5}}{1}$$

$$\boxed{L = 5 \times 10^{-3} \ H}$$

Ques Calculate coefficient of self inductance of 1 m long solenoid of 500 turns and 5 cm diameter.

$N = 500$ turns

$r = \frac{5}{2}$ cm $= \frac{5}{200}$ m $= \frac{1}{40}$ m

$l = 1$ m

through it changes by 250 mA in 0.1 sec ? Calculate emf induced in the

$$L = \frac{\mu_0 N^2 A}{l}$$

$$L = \frac{4 \times 3.14 \times 10^{-7} \times 500 \times 500 \times \pi r^2}{1}$$

$$L = 0.62 \times 10^{-3} \ H$$

$$\boxed{L = 0.62 \ mH}$$

Expression for Energy Stored in an Inductive circuit and Energy density :

When current is passed in an inductor, back emf i.e $-L\frac{di}{dt}$ induced in it & opposes the growth of current. Thus to grow current and take desired value i_0, work is done by the battery. This work is stored as mag. field lines in inductor.

Small work done in moving charge dq against back emf $\left(-L\frac{di}{dt}\right)$ by battery

$$dw = emf \times charge$$

$$dw = \left(-L\frac{di}{dt}\right) \times dq$$

$$= -L\left(\frac{dq}{dt}\right) \cdot di$$

Page No.
Date : / /201

$$\frac{dq}{dt} = i$$

$$dw = -L\,i\,di$$

(−) means oppose.

$$dw = L\,i\,di$$

On integration

$$W = \int_0^{i_0} dw$$

$$W = \frac{L i_0^2}{2}$$

$$W = \frac{1}{2} L i_0^2$$

This work stored in inductor as M.F.L.

$$\boxed{U = \frac{1}{2} L i_0^2}$$

Now,
we calculate energy density in following way

$$U = \frac{1}{2} L i_0^2$$

where $L = \frac{\mu_0 N^2 A}{l}$

$\therefore \quad U = \frac{1}{2} \frac{\mu_0 N^2 A}{l} \cdot i_0^2$

Also

$$B = \mu_0 n i_0 = \frac{\mu_0 N i_0}{l}$$

$$\Rightarrow i_0 = \frac{B l}{\mu_0 N}$$

so

expression of energy becomes

$$U = \frac{1}{2} \left(\frac{\cancel{\mu_0} \cancel{N^2} A}{\cancel{l}} \right) \times \frac{B^2 l^2}{\mu_0^2 \cancel{N^2}}$$

$$U = \frac{1}{2} \frac{B^2 (A \times l)}{\mu_0}$$

$$A \times l = \text{volume} = V$$

$$U = \frac{1}{2} \frac{B^2}{\mu_0} V \cdot m^3$$

energy per m^3.

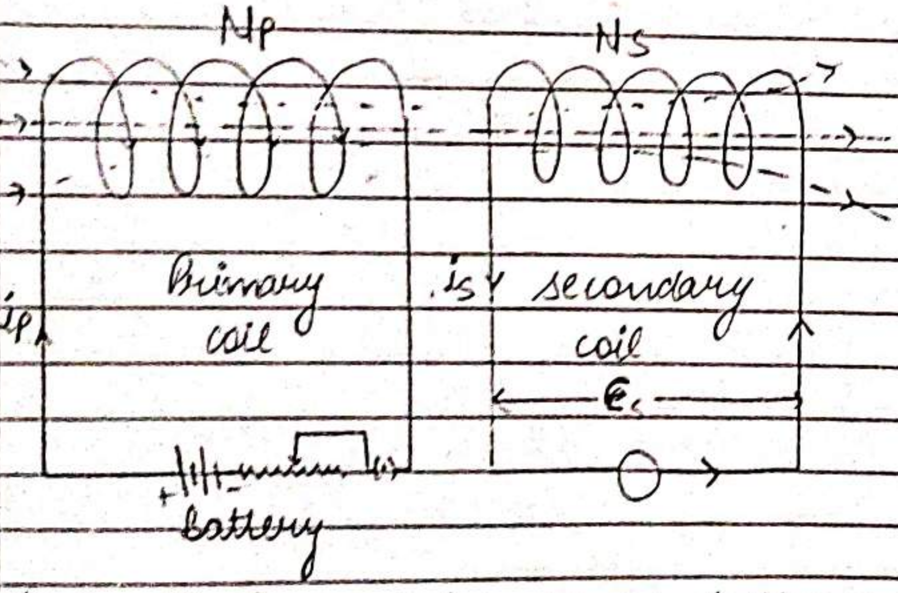
$$u = \frac{U}{V}$$

$u = \frac{1}{2} \frac{B^2}{\mu_0} \frac{J}{m^3}$ it is called energy density.

$$\boxed{u = \frac{B^2}{2\mu_0}}$$

* Mutual Induction

When current passing through a coil is changed, then magnetic flux linked with neighbouring coil also change so an induced emf is set up in the secondary coil. This phenomenon is called Mutual Induction.



The coil in which current changes is called Primary coil whereas the other in which emf is produced is called secondary coil.

Page No.
Date:

Mutual Inductance or coefficient of M

Let N_s = no. of turns in secondary coil
ϕ_s = flux linked with each turn

$N_s\phi_s$ = total flux linked with secondary or magnetic flux linkages.

Experimentally,

$$N_s\phi_s \propto i_p$$

or $N_s\phi_s = M i_p$

M is called Mutual inductance b/w two coils.

$$\boxed{M = \frac{N_s\phi_s}{i_p}} \quad \text{--- (1)}$$

acc. to Faraday
emf induced in secondary coil.

$$E_s = -N_s \frac{d\phi_s}{dt}$$

$$E_s = \frac{-d}{dt}(N_s\phi_s)$$

put $N_s\phi_s = M i_p$

$$E_s = \frac{-d}{dt}(M i_p)$$

Page No.
Date : / /201

$$E_s = -M \frac{di_p}{dt}$$

$$M = \frac{-E_s}{\left(\frac{di_p}{dt}\right)}$$

or

numerically

$$\boxed{M = \frac{E_s}{\left(\frac{di_p}{dt}\right)}} \qquad \text{———} (2)$$

definition of M :

From eqn (1) —

"Mutual inductance b/w two coils is numerically equal to no. of magnetic flux linkages of secondary coil when unit current passed through primary coil."

From eqn (2) —

"Mutual inductance b/w two coils is numerically equal to induced emf in one coil, when rate of change of current is kept unity in neighbouring coil"

Eg. Transformer is a device which works on mutual induction.

7.5(a) (2013)

Combination of Two Inductors

a) In series b) In parallel

(a) <u>In series</u> : When two coils are <u>distant</u> and connected in series, a current i is passed through them.

Flux linked with first coil

$$\phi_1 = L_1 i$$

With second coil $\phi_2 = L_2 i$

If their resultant inductance is L then total flux,

$$\phi_{total} = Li$$

Now,

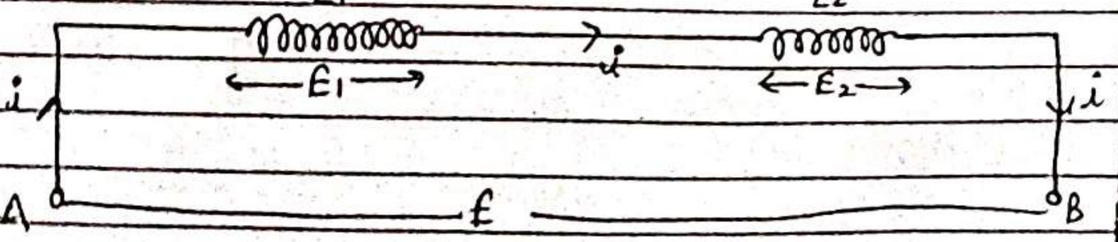
$$\phi_{total} = \phi_1 + \phi_2$$

$$Li = L_1 i + L_2 i$$

$$L\cancel{i} = (L_1 + L_2)\cancel{i}$$

$$\boxed{Li = (L_1 + L_2)i}$$

$$\boxed{\phi_{total} = (L_1 + L_2)i}$$



Page No.
Date : / /201

Now, when the coils are separated by a small distance then there will be a mutual inductance 'M' b/w them then

$$E_1 = -L_1\frac{di}{dt} - M\frac{di}{dt}$$

$$E_2 = -L_2\frac{di}{dt} - M\frac{di}{dt}$$

& $$E = -L_s\frac{di}{dt}$$

put in $E = E_1 + E_2$

$$-L_s\frac{di}{dt} = (-L_1 - M - L_2 - M)\frac{di}{dt}$$

$$-L_s = -L_1 - L_2 - 2M$$

$$\boxed{L_s = L_1 + L_2 + 2M}$$

here it is assume coils are so placed that flux linking with each coil due to current is in same direction and support.

If coil are so placed that flux of both opposess then

$$\boxed{L_s = L_1 + L_2 - 2M}$$

In general,

$$\boxed{L_s = L_1 + L_2 \pm 2M}$$

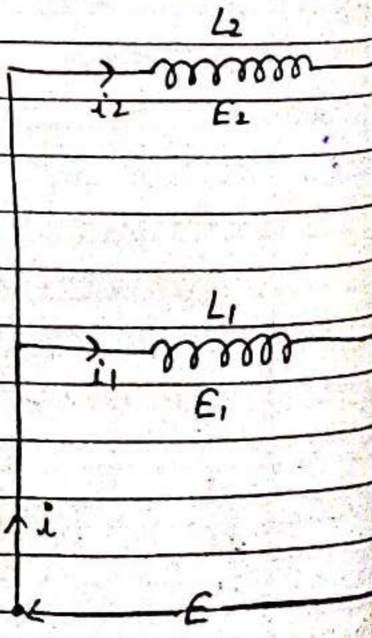
b) In Parallel: Let two coils of self inductances L_1 and L_2 are connected in parallel b/w two points and current i flowing in them and divide $i = i_1 + i_2$

On differentiating

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \quad \text{——— (1)}$$

When current through inductances will grow, induced emf were set up in both. So $E_1 = E_2 = E$.

By Faraday



$$E = -L_1 \frac{di_1}{dt}$$

$$\frac{di_1}{dt} = -\frac{E}{L_1}$$

& $E = -L_2 \frac{di_2}{dt}$

$$\frac{di_2}{dt} = -\frac{E}{L_2}$$

$$E = -L_p \frac{di}{dt}$$

$$\& \quad \frac{di}{dt} = \frac{-E}{L_p}$$

So, substituting in eqⁿ ①

$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} \qquad L_p = \frac{L_1 L_2}{L_1 + L_2}$$

Or $$\boxed{L_p = \frac{L_1 L_2}{L_1 + L_2}}$$

This relation holds good when both inductors are seperated by large distances $(M=0)$

If the separation is small, then

$$\boxed{L_p = \frac{L_1 L_2 - M^2}{L_1 + L_2 \pm 2M}}$$

Ques. Two inductances L_1 and L_2 are connected:
(i) in series.
(ii) in parallel
and are separated by a large distance. Find the equivalent inductance in each case. How will the result be affected if the separation is not large?